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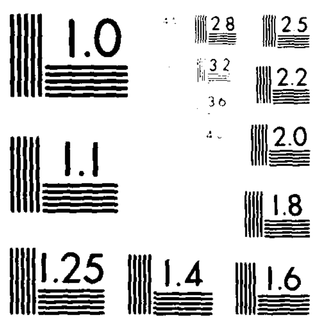

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THE NORMAL MODES OF BEAM-LIKE STRUCTURES BY A LANCZOS-STODOLA METHOD

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SUMMARY

A method of finding the flexural and torsional normal modes of structures which have straight stiffness axes and which are mounted as cantilevers is given. The Lanczos method of minimised iterations is used to obtain intermediate modes using the integro-differential equations and the mass and stiffness distributions. The inertia matrix appropriate to the intermediate modes is tridiagonal and the stiffness matrix is unit. The dominant eigenvalues and vectors of the inertia matrix give good approximations to the graver normal modes. Results of test calculations using a computer program which also allows for the presence of discrete masses are given.

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1 INTRODUCTION

In the early stages of the design of wings of moderate-to-large aspect ratios their structural properties are often given in terms of spanwise mass and stiffness distributions, the stiffness properties being taken to be concentrated at a straight spanwise stiffness axis. The normal modes of such wings, mounted as cantilevers, are needed for investigations of their oscillatory characteristics and aeroelastic behaviour and a method is described herein whereby these modes can be obtained directly from the distributions of mass and stiffness without the intermediate step of either replacing the continuous mass of the beam by discrete masses and moments of inertia and the stiffness by a flexibility matrix with a consequent loss of accuracy or choosing an adequate set of assumed modes.

The method is based on the numerical solution of the integro-differential equations describing flexural and torsional oscillations (equation (6)). Without the help of a digital computer such solutions are tedious, calling as they do for the evaluation of numerous integrals. A digital computer removes the tedium; and accuracy can be achieved by integrating for a large number of spanwise stations even if the method of numerical integration used is the simplest. The direct iterative solution of the equations, Stodola's method[1], breaks down when the frequencies of two or more of the modes are close together in just the same way as does its equivalent, the power method of finding the eigenvalues of matrices. A possible way of avoiding this, whilst retaining the valuable properties of Stodola's method, is to find a set of intermediate modes which are orthogonal to each other with respect to stiffness by Stodola iteration and then to solve the eigenvalue equation which has the inertia and stiffness matrices of these modes as coefficients. Not only will the stiffness matrix be diagonal but the inertia matrix will be tridiagonal for this is the Lanczos method of minimised iterations[2]. For convenience the intermediate modes will be referred to as Lanczos modes. They can be looked upon as a set of assumed modes with special properties.

The first Lanczos modes are composed predominantly of the graver normal modes just as the first mode found by Stodola's method is the fundamental. In fact Lanczos noticed that, if the frequencies of the normal modes are well-separated, the inclusion of an extra Lanczos mode adds one more normal mode to the solution and corrects the graver normal modes by only small amounts. In general the frequencies are not as well-separated as this but the economy that can be exercised in the number of Lanczos modes used is the principal attraction

of the method. Were it not possible to get good results with a restricted number of Lanczos modes the method would not be used because as the number of Lanczos modes calculated is increased cancellation results in a crippling loss of accuracy[3].

In the following sections an iterative procedure for transforming the integro-differential equations into a tridiagonal-matrix equation is derived from an established procedure for transforming the full-matrix eigenvalue equation and some details of the computer program that has been written are given. Wings often carry discrete masses and, because these cannot be represented accurately in the simple methods of integration used, provision is made in the program for the inclusion of discrete-load modes. These are particularly useful in investigations in which the sizes of the discrete masses are varied[4]. Finally the results of tests to establish the range of applicability of the program are given.

2 THE ITERATIVE PROCEDURE

2.1 The equivalent matrix equation

The basis of the method, described in section 2.2 below, is derived from that used for finding the dominant eigenvalues of the equivalent matrix equation which can be written

$$(A - \lambda E)q_a = 0 \qquad (1)$$

where A and E are square symmetric matrices of structural inertia and stiffness coefficients, $\lambda = \omega^{-2}$, ω is the frequency of a normal mode and q_a is its modal vector.

The Lanczos method of minimised iteration can be applied to find n intermediate modes by the following sequence of operations[5]:

$$\left.\begin{array}{ll}
(1) & \text{Take a random vector, } \bar{q}_1 \text{, and put } i = 1 \text{ ;} \\
(2) & \text{Put } \beta^2_{i-1} = \bar{q}'_i E \bar{q}_i \text{ ;} \\
(3) & \text{Normalise by putting } q_i = \beta^{-1}_{i-1}\bar{q}_i \text{ ;} \\
(4) & \text{Find the direct inertia coefficient, } \alpha_i = q'_i A q_i \text{ ;} \\
(5) & \text{Iterate to obtain next raw vector, } \bar{\bar{q}}_{i+1} = E^{-1}Aq_i \text{ ;} \\
(6) & \text{Orthogonalise with respect to stiffness, } \bar{q}_{i+1} = \bar{\bar{q}}_{i+1} - \alpha_i q_i - \beta_{i-1}q_{i-1} \\
 & \text{where } \beta_0 q_0 = 0 \text{ ;} \\
(7) & \text{Add 1 to } i \text{ and repeat from (2) until required number, } n \text{, of } q_i \\
 & \text{have been obtained when stop at (4).}
\end{array}\right\} \qquad (2)$$

A dash in the above signifies transposition.

It has been shown[2,3,5] that

$$Q'AQ \;=\; \bar{A} \tag{3}$$

where $Q = (q_1, q_2, \ldots, q_n)$,
and $\bar{A}$ is a symmetric tridiagonal matrix with leading diagonal $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ and sub- and super-diagonals each $(\beta_1, \beta_2, \ldots, \beta_{n-1})$.

In the sequence (2) the q_i have been normalised so that $Q'EQ = I$ and so the solutions of

$$(\bar{A} - \lambda I)q_b \;=\; 0 \tag{4}$$

are approximations to the dominant eigenvalues of equation (1). The eigenvectors, q_a , are then given by

$$q_a \;=\; Qq_b \quad . \tag{5}$$

The order, n , of the matrices in equation (4) need only be about twice the number of eigenvalues required.

2.2 The integro-differential equations

The integro-differential equation which governs a wing vibrating in flexure and torsion can be written

$$\begin{bmatrix} \zeta''(\eta) \\ \\ \theta'(\eta) \end{bmatrix} = \omega^2 \begin{bmatrix} \phi_{\zeta\zeta}(\eta)\phi_{\zeta\theta}(\eta) \\ \\ \phi_{\theta\zeta}(\eta)\phi_{\theta\theta}(\eta) \end{bmatrix} \begin{bmatrix} \displaystyle\int_\eta^1 d\eta_1 \int_{\eta_1}^1 \{\mu(\eta_2)\zeta(\eta_2) + \overline{\mu\xi}(\eta_2)\theta(\eta_2)\} d\eta_2 \\ \\ \displaystyle\int_\eta^1 \{\overline{\mu\xi}(\eta_2)\zeta(\eta_2) + \overline{\mu\kappa}^2(\eta_2)\theta(\eta_2)\} d\eta_2 \end{bmatrix} \tag{6}$$

where η is a spanwise coordinate, zero at the root and unity at the tip,
$\zeta(\eta)$ is a scaled heave displacement,
$\theta(\eta)$ is an incidence displacement,
$\phi_{\zeta\zeta}(\eta)$, $\phi_{\theta\theta}(\eta)$, $\phi_{\zeta\theta}(\eta)$, $\phi_{\theta\zeta}(\eta)$ are the direct and cross-flexibilities,
$\mu(\eta)$ is the line density of the mass,
$\overline{\mu\xi}(\eta)$ is the line density of the first moment of mass about the stiffness axis,
$\overline{\mu\kappa}^2(\eta)$ is the line density of the second moment of mass about the local centre of gravity,
and dashes here signify differentiation with respect to η .

The 2×2 matrix of flexibilities includes crossflexibilities so that account may be taken of the special properties of fibre-reinforced plastics with asymmetric lay-ups.

The essentials of the method can be demonstrated by considering only heave and so the equation considered is

$$\zeta'' = \omega^2 \phi_{\zeta\zeta} \psi_\zeta \tag{7}$$

where ψ_ζ is the bending moment distribution, the repeated integral in equation (6). For our immediate purposes the suffices ζ can be omitted.

In the Stodola method the sequence of operations is:

(1) Choose a deflection mode ζ containing components of all the graver normal modes and put $i = 1$;

(2) Put $\bar{\zeta}_i^{(1)} = \zeta$;

(3) Make the mode orthogonal to previous modes with respect to mass by putting $\zeta_i^{(j)} = \bar{\zeta}_i^{(j)} + \sum_{k=1}^{i-1} \kappa_k \zeta_k$ where the κ_k are chosen to make $\int_0^1 \mu \zeta_i^{(j)} \zeta_k d\eta = 0$ ($i > 1$ only);

(4) Find the bending moment distribution, $\psi_i^{(j)} = \int_\eta^1 d\eta_1 \int_{\eta_1}^1 \mu \zeta_i^{(j)} d\eta_2$;

(5) Find the curvature distribution, $\zeta_i''^{(j+1)} = \phi \psi_i^{(j)}$; (8)

(6) Integrate $\zeta_i''^{(j+1)}$ twice to obtain the deflection mode $\bar{\bar{\zeta}}_i^{(j+1)}$ and normalise by putting $\bar{\zeta}_i^{(j+1)} = \left(\omega_i^{(j+1)}\right)^2 \bar{\bar{\zeta}}_i^{(j+1)}$, where $\omega_i^{(j+1)}$ is determined by the condition $\left(\bar{\zeta}_i^{(j+1)}\right)_{tip} = 1$;

(7) Test for convergence. If $\int_0^1 \left(\bar{\zeta}_i^{(j+1)} - \zeta_i^{(j)}\right)^2 d\eta > \varepsilon$, a suitable small number related to the numerical accuracy of the computer, add 1 to j and go to (3); otherwise, accept the normal mode, $\zeta_i = \bar{\zeta}_i^{(j+1)}$, and natural frequency, $\omega_i = \omega_i^{(j+1)}$, add 1 to i and go to (2).

In the proposed method, following (2), the sequence of operations is:

(1) Choose an initial moment distribution, $\bar{\psi}_1$, and put $i = 1$;

(2) Find the curvature distribution, $\bar{\zeta}''_i = \phi\bar{\psi}_i$, and put $\beta^2_{i-1} = \int_0^1 \bar{\psi}_i\bar{\zeta}''_i d\eta$;

(3) Put $\psi_i = \beta^{-1}_{i-1}\bar{\psi}_i$, $\zeta''_i = \beta^{-1}_{i-1}\bar{\zeta}''_i$;

(4) Integrate ζ''_i twice to obtain ζ_i and put $\alpha_i = \int_0^1 \mu\zeta_i^2 d\eta$; (9)

(5) Find the bending moment distribution, $\bar{\bar{\psi}}_{i+1} = \int_\eta^1 d\eta_1 \int_{\eta_1}^1 \mu\zeta_i d\eta_2$;

(6) Make it orthogonal to distributions in previous modes,

$\bar{\psi}_{i+1} = \bar{\bar{\psi}}_{i+1} - \alpha_i\psi_i - \beta_{i-1}\psi_{i-1}$, where $\beta_0\psi_0 = 0$;

(7) Add 1 to i, repeat from (2) until required number of ζ_i have been obtained when stop at (4).

All the operations are numerical, of course, and the variables ϕ, ψ, ζ, etc are represented by their values at a large number of spanwise stations. Again the α_i and β_i form the leading diagonal and sub- and super-diagonal of the matrix $\bar{A}$ in equation (4). Similarly the eigenvectors are given by

$$\zeta_a = Zq_b \quad . \tag{10}$$

where $Z = (\zeta_1, \zeta_2, \ldots, \zeta_n)$, and here the ζ_i are column vectors of the values of ζ .

At stage (6) in each sequence the orthogonality to previous modes can deteriorate catastrophically due to cancellation[3] and it is necessary that $\bar{\psi}_{i+1}$ (in the present case) be re-orthogonalised. This can be done by the sequence:

$$\left.\begin{aligned} \bar{\zeta}''^{(r)}_{i+1} &= \phi\bar{\psi}^{(r)}_{i+1} \\ \chi^{(r)}_j &= \int_0^1 \psi_j\bar{\zeta}''^{(r)}_{i+1} d\eta \\ \bar{\psi}^{(r+1)}_{i+1} &= \bar{\psi}^{(r)}_{i+1} - \chi^{(r)}_j\psi_j \end{aligned}\right\} \tag{11}$$

the sequence being repeated with the same value of j until χ_j is smaller than some number ε whose size depends upon the numerical precision of the computer. $\bar{\psi}_{i+1}$ must be reorthogonalised to all ψ_j , $j = 1, i$.

3 SOME DETAILS OF THE PROGRAM

The part-span integrals are evaluated by the trapezium rule and full-span integrals by Simpson's rule.

The initial moment distribution, $\bar{\psi}_1$, taken is that due to the deflection of the wing under a combination of discrete heave load and torsional moment at $\frac{3}{4}$ span. This is in the spirit of the recommendation of Lanczos (Ref 2, p.270) that the initial vector should be an arbitrary vector multiplied twice by the dynamical matrix.

The value of the accuracy criterion, ε , is obtained by factoring a user-supplied value by an approximation to the next value of β so that the criterion is one of relative accuracy rather than absolute accuracy.

Normal modes are calculated by using the procedure *tql2* of Ref 6.

Discrete-load modes are included after the normal modes of the bare wing have been calculated. In their initial form they are the modes for discrete force in heave and moments in roll and pitch at the loaded station. Thus three extra modes are included for each loaded station. The modes are subsequently made orthogonal, with respect to stiffness, to each other and to the normal modes of the bare wing. The orthogonalisation used is that of sequence (11). It is repeated because the initial discrete-load modes will be similar in shape to the bare-wing modes already included. The modes are normalised so that their stiffness coefficients are unity. The inertia matrix is transformed to tri-diagonal by the procedure *tred2* of Ref 6.

4 RESULTS OF TEST CALCULATIONS

The accuracy of an approximate normal mode is usually judged by the difference between its frequency and the exact frequency. When the data for a normal-mode calculation are in the form of assumed modes whose inertia and stiffness coefficients are known exactly the approximate frequencies will be equal to or higher than the exact frequencies and generally the percentage error will increase motonically with the order of the mode. This was often the case in the present calculations but not always. Sometimes the error decreased after first increasing and in these cases the mode at which the decrease first occurred and higher-order modes were considered inaccurate. Another indication of

numerical inaccuracy in the formation of the Lanczos modes was the appearance of spurious modes so that two modes appeared with the same characteristics at different frequencies. Higher-order modes (and, of course, the spurious modes) were considered inaccurate.

4.1 Bare wing

The majority of test calculations were made for a uniform beam because the exact natural frequencies are known. There were two program variables to investigate: one was the number of spanwise elements used in the numerical integration and the other was the number of Lanczos modes calculated. The amount of computer store needed is roughly proportional to the product of these numbers and it is important to know which to reduce if storage is limited.

Calculations were made for a beam in flexure and the results are given in Table 1. Within the range covered the accuracy of the modal frequencies increases as the number of spanwise elements is increased but there is a limit

Table 1

Uniform beam in flexure

No. of integration elements	No. of Lanczos modes calculated	No. of normal modes found with frequency error less than:	
		1%	0.1%
500	9	8	7
400	11	9	8
300	15	12	7
	9	8	7
200	22	6	5
	15	10	5
100	22	7	2
	15	8	2
	9	7	2

after which further increases do not seem worthwhile. When the number of Lanczos modes calculated is increased, the number of modes found to the higher accuracy does not increase. However, the number of modes found to the lower accuracy does vary with the number of Lanczos modes. For 300 elements the number found to the lower accuracy increases with the number of Lanczos modes but with lesser numbers of elements the number of normal modes found decreases

as the number of Lanczos modes is increased. In both the 200- and 100-element cases, when the number of Lanczos modes was 22 the sequence of good modes was terminated by a spurious mode. In both cases, when the number of Lanczos modes was 15, the limit was taken as the highest mode whose frequency error was greater than that of the previous mode. From these results it is more important to take a large number of elements. In fact it is dangerous to extract a large number of Lanczos modes because of the effect of cancellation which the results for 22 modes illustrate. The number of accurate modes found will obviously depend on the numerical precision in the computer. All the present calculations were made using a computer in which the mantissa is 37 bits in length. A greater number of accurate modes will be found *in toto* if there is torsion as well as flexure. A calculation with both flexure and torsion, on a beam described below, gave 15 satisfactory normal modes from 200 elements and 22 Lanczos modes.

Calculations of the torsion modes with the beam having 300 elements gave the results of Table 2. The modal displacements were within about 1% of the exact values. To test the program on a non-uniform beam the graver flexural

Table 2

Uniform beam in torsion, 300 elements, frequency error <0.1%

No. of Lanczos modes	15	9	5
No. of normal modes	10	5	2

normal modes of truncated cones with tip diameters 0.4 and 0.5 of the root diameter were calculated and compared with the reference values provided by Downs[7]. These calculations were made using 500 elements and nine Lanczos modes. In each case the first seven modes were within 0.1% on frequency. The fundamental frequencies found were below those given by Downs but within 0.01%.

To test the ability of the program to deal with modes close in frequency the normal modes of a uniform beam with both flexure and torsion allowed were calculated. There was no coupling of flexure and torsion and the exact frequency of the fundamental torsion mode was within 0.01% of the exact frequency of the first overtone flexure mode. The results of the calculation are given in Table 3. The criterion for acceptability in this case was the quality of the orthogonality of flexure and torsion. Because there was no coupling between flexure and torsion the normal modes calculated should have been either

Table 3

Uniform beam with close frequencies in flexure and torsion, 300 elements

No. of Lanczos modes	15	13	11	9	8	7	6	5
No. of normal modes	9	8	8	5	5	5	1	1

pure flexure or pure torsion. However calculated flexural normal modes contained some torsion and *vice versa* but the impurity could be satisfyingly small. There was always a distinct division between modes with satisfactory and unsatisfactory orthogonality and the frequency error of the modes that were acceptable was less than 0.05%. It is noticeable that the number of approximate normal modes does not increase regularly as the number of Lanczos modes is increased. In this example not only are the second and third frequencies almost the same but the fourth and fifth frequencies are within 7% of each other and the sixth and seventh frequencies are within 10% of each other and so the observation of Lanczos on systems with widely-spaced frequencies to the effect that another normal mode is obtained for every intermediate mode added to the calculation does not apply.

4.2 Loaded wing

Test calculations were made for a loaded wing to prove that part of the program in which discrete-load modes are involved. In the tests very large masses and moments of inertia were put at the tip so that the overtone modes could be compared with the modes of a propped cantilever and built-in beam. In all the calculations 200 elements, eight Lanczos modes and five bare-wing normal modes were used.

For only a mass at the tip (propped cantilever) the first five frequencies of the loaded wing were found to within 0.1%. For both mass and rolling inertia at the tip (built-in beam) the first four frequencies were found to within 1%. For only pitching inertia at the tip (built-in beam) the first four torsional frequencies were found to within 0.1%. These test cases are severe and show that good results can be obtained even when the numbers of integration elements and bare-wing modes used are not large.

4.3 Comments on use

From the results obtained the method should give good approximations to as many of the graver normal modes as are needed in aeroelastic investigations.

In view of the results in Table 3 it is probably wise to use at least seven intermediate modes but it is quite possible that five good normal modes can be obtained from these seven intermediate modes if the beam is divided up into an adequate number of elements. If five or more modes are needed however it is better to use more intermediate modes but there seems to be little point in using even as many as twice the number of normal modes required. Indeed, if the number of normal modes required is large, some restraint needs to be shown in the number of intermediate modes used if numerical difficulties are to be avoided but the position is eased if the beam can distort in both flexure and torsion.

Little mention has been made of the accuracy of the shape of the calculated mode because evaluating it is difficult. However, the calculated modes were studied and it is thought that the maximum error in deflection will be of the order of 1% for a mode with a frequency error of less than 0.1%.

5 CONCLUSIONS

The Lanczos method of minimised iterations has been developed to calculate the graver normal modes of cantilever beam-like structures directly from the distributions of their mass and stiffness. The method is not one for purely mechanical application but the computer program written has given accurate results for as many modes as will be required in most practical cases. No definite rules are given for the number of intermediate modes that should be used because this will depend on the numerical precision of the computer being used. However illustrative results have been found which indicate the relative merits of having a large number of integration elements and a large number of intermediate modes. The program also includes provision for discrete-load modes and the application of these has been demonstrated by calculating the normal modes of beams carrying large concentrated masses.

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17. Abstract

A method of finding the flexural and torsional normal modes of structures which have straight stiffness axes and which are mounted as cantilevers is given. The Lanczos method of minimised iterations is used to obtain intermediate modes using the integro-differential equations and the mass and stiffness distributions. The inertia matrix appropriate to the intermediate modes is tridiagonal and the stiffness matrix is unit. The dominant eigenvalues and vectors of the inertia matrix give good approximations to the graver normal modes. Results of test calculations using a computer program which also allows for the presence of discrete masses are given.

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